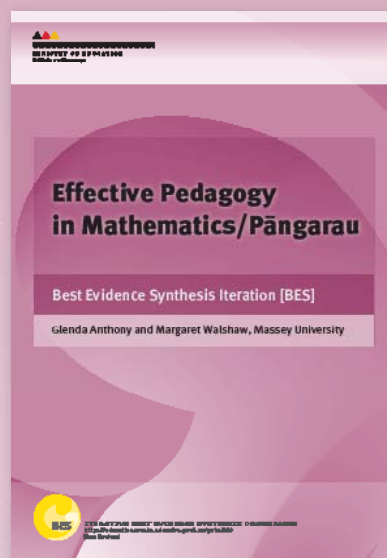


# Strengthen teacher pedagogical knowledge

This is one of a series of cases that illustrate the findings of the best evidence syntheses (BESs). Each is designed to support the professional learning of educators, leaders and policy makers.



## BES cases: Insight into what works

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The best evidence syntheses (BESs) bring together research evidence about ‘what works’ for diverse (all) learners in education. Recent BESs each include a number of cases that describe actual examples of professional practice and then analyse the findings. These cases support educators to grasp the big ideas behind effective practice at the same time as they provide vivid insight into their application.

Building as they do on the work of researchers and educators, the cases are trustworthy resources for professional learning.

### Using the BES cases

The BES cases overview provides a brief introduction to each of the cases. It is designed to help you quickly decide which case or cases could be helpful in terms of your particular improvement priorities.

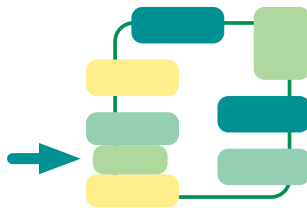
Use the cases with colleagues as catalysts for reflecting on your own professional practice and as starting points for delving into other sources of information, including related sections of the BESs. To request copies of the source studies, use the Research Behind the BES link on the BES website.

The conditions for effective professional learning are described in the Teacher Professional Learning and development BES and condensed into the ten principles found in the associated International Academy of Education summary (Timperley, 2008).

Note that, for the purpose of this series, the cases have been re-titled to more accurately signal their potential usefulness.

### Responsiveness to diverse (all) learners

Use the BES cases and the appropriate curriculum documents to design a response that will improve student outcomes



The different BESs consistently find that any educational improvement initiative needs to be responsive to the diverse learners in the specific context. Use the inquiry and knowledge-building cycle tool to design a collaborative approach to improvement that is genuinely responsive to your learners

### Strengthen teacher pedagogical knowledge

Using fractions as a context, these two cases highlight the importance of teachers being able to create conceptually correct representations of mathematical ideas. Without the knowledge to do this, attempts to connect mathematics with students’ lives are always going to fall short.

The examples in these cases provide a springboard for teachers to discuss their own mathematical understandings with colleagues or in a professional learning community. This process could highlight areas that need addressing.

See also BES Case 5: *Use mathematical tools to explore students’ thinking about mathematics.*

## Teacher knowledge: Forms of ‘knowing’ fractions

In all of the CASEs, the centrality of teacher knowledge is evident. Reiterating discussions from earlier chapters, what teachers do is very dependent on what they understand about the teaching and learning of mathematics. For fractions, in particular, knowing different models and various approaches to the teaching of fractions places high demands on teachers’ mathematical and pedagogical content knowledge.

Unfortunately, numerous studies point to shortcomings in teachers’ understanding of rational numbers (e.g., Domoney, 2001). In a seminal study comparing US and Chinese teachers’ mathematical knowledge, Ma (1999) demonstrated how US teachers more readily situated fraction problems in real-world contexts. However, this apparent familiarity with and link to everyday experience appeared to be superficial. Ma found substantial differences in knowledge when teachers were asked to perform division of fractions or to generate representations of fractions. For example, only one of the 23 US teachers in the study generated a conceptually correct representation for the meaning of the equation  $1\frac{3}{4} \div \frac{1}{2}$ . This compares with 65 of the 72 Chinese teachers. Among the 23 US teachers, 6 could not create a story to match the calculation and 16 provided stories that contained misconceptions. Twelve of the misconceptions involved confusing division by  $\frac{1}{2}$  with division by 2 or multiplication by  $\frac{1}{2}$ . For example: *Jose has one and three-fourths boxes of crayons and he wants to divide them between two people or divide the crayons in half, and then, first we could do it with crayons and maybe write it on the board or have them do it in numbers.* Ma cautioned that although US teachers reported the frequent use of real contexts, the ‘real world’ cannot produce the mathematical content by itself. She claims that “without a solid knowledge of what to present, no matter how rich one’s knowledge of students’ lives, no matter how much one is motivated to connect mathematics with students lives, [this is without benefit] if one still cannot produce a conceptually correct representation” (p. 82).

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### CASE 8: Representation of Division by Fractions

(from Ma, 1999)

All of the Chinese teachers successfully computed  $1\frac{3}{4} \div \frac{1}{2}$  and 65 of the 72 created a total of more than 80 story problems representing the meaning of division by a fraction. The Chinese teachers represented the concept using three different models of division: measurement (quotitive), partitive (sharing), and product and factors. For example,  $1\frac{3}{4} \div \frac{1}{2}$  might represent:

- $1\frac{3}{4}$  metres  $\div$   $\frac{1}{2}$  metre =  $7\frac{1}{2}$  (quotitive model)
- $1\frac{3}{4}$  metres  $\div$   $\frac{1}{2}$  =  $7\frac{1}{2}$  metres (partitive model)
- $1\frac{3}{4}$  square metres  $\div$   $\frac{1}{2}$  metre =  $7\frac{1}{2}$  metres (product and factors)

corresponding to the problems:

- How many  $\frac{1}{2}$  m lengths of timber are there in  $1\frac{3}{4}$  m of timber?
- If half a length of timber is  $1\frac{3}{4}$  m, how long is the whole piece of timber?
- If one side of a  $1\frac{3}{4}$  square metre rectangle is  $\frac{1}{2}$  m, how long is the other side?

In their discussions of the meaning of division by fractions, the Chinese teachers mentioned several concepts that they considered related to the topic. These are represented in the diagram:

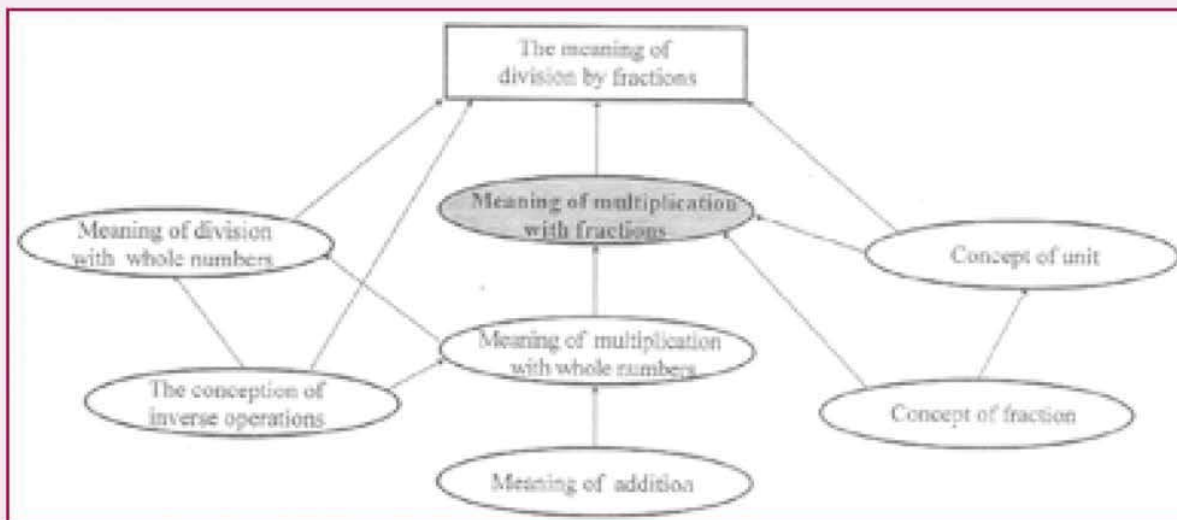


Fig. 7.9. Connected knowledge for understanding the meaning of division

Their view of connected knowledge translated into a forward trajectory of learning. Work on division by fractions was also valued for the role in intensifying concepts of rational number already encountered by the students. The Chinese teachers expressed the view that students may “gain new insight through reviewing old ones [concepts]. The current learning is supported by, but also deepens, the previous learning” (p. 77).

A more recent comparative study of US and Chinese teachers (Shuhua, Kulm, & Wu, 2004) also notes the different system demands on teachers’ pedagogical content knowledge. The researchers express concern about the pedagogical approaches in the US system that indicate a “lack of connection between manipulative and abstract thinking, and between understanding and procedural development” (p. 170).

As we have seen earlier, teacher knowledge is also a significant factor in the interpretation of students’ thinking and solution strategies. For example, in assessing students’ understanding of fractions, sound pedagogical content knowledge is needed to determine the effectiveness of different tasks.

## CASE 9: Knowledge of Students Solving Fractions

(from Grossman, Schoenfeld, and Lee, 2005)

Which of the following tasks would best assess whether a student can correctly compare fractions?

- Write these fractions in order of size, from smallest to largest:  $\frac{5}{8}$ ,  $\frac{1}{4}$ ,  $\frac{1}{16}$
- Write these fractions in order of size, from smallest to largest:  $\frac{5}{8}$ ,  $\frac{3}{4}$ ,  $\frac{1}{16}$
- Write these fractions in order of size, from smallest to largest:  $\frac{5}{8}$ ,  $\frac{3}{4}$ ,  $\frac{1}{16}$

Can you explain why the two tasks you did not select are not good assessments of students’ understanding of fractions?

To do this, teachers need to know the relevant mathematics in a deep and connected way (Ma, 1999). Teachers need more than just the ability to solve the problem. They also need to know about the ways in which students might solve the problem and the reasoning that they may or may not use. For example, there are at least two ways to solve the problem: converting the fractions to decimals and comparing them or reasoning through the task by comparing the fractions themselves. Of the three fractions in the first set, only one,  $\frac{1}{4}$ , is less than  $\frac{1}{2}$ . So  $\frac{1}{4}$  is the smallest. And then because  $\frac{5}{8} = \frac{10}{16}$ , and  $\frac{10}{16}$  is less than  $\frac{11}{16}$ ,  $\frac{5}{8}$  is less

than  $\frac{1}{16}$ . Thus the order is  $\frac{1}{4}$ ,  $\frac{5}{8}$ ,  $\frac{1}{16}$ . But the real issue for the teacher is how their own students will solve this problem.

Research shows that many students will focus only on the number of pieces, not their relative size. For example, given the first set of three fractions, students will think, " $\frac{1}{4}$  has only one piece, so it's the smallest;  $\frac{5}{8}$  has five pieces, so it's in the middle; and  $\frac{1}{16}$  has eleven pieces, so it's the largest." Unfortunately this incorrect reasoning produces the right answer. Another common misconception held by students is that "the smaller the pieces, the smaller the fraction." Because sixteenths are smaller than eighths and eighths are smaller than fourths, students using this reasoning may arrive at the correct answer, given the second set of three fractions. In the case of the third set of three fractions, however, students who use either of these forms of incorrect reasoning will get the wrong answer—and their wrong answer will suggest why they got it wrong.

With fractions, as with other areas of mathematics, teachers need to distinguish what the student understands as opposed to what the student can do (Pearn & Stephens, 2004). The teacher may need to listen across multiple tasks in order to determine how a student is thinking. The following response by Madison, a student in Mitchell and Clarke's (2004) research, illustrates the value of multiple tasks in the diagnostic setting. When Madison was asked to respond to an estimation task involving adding pairs of fractions near 1 and near  $\frac{1}{2}$ , her estimate for  $\frac{7}{8} + \frac{12}{13}$  was "two"—the correct answer. According to the researchers, her reasoning appeared modest but faultless: "I just guessed. That's seven bits of eight. Twelve bits of thirteen." Based on the assumption that Madison had used a part-whole approach, her follow-up comment seemed strange and unrelated to her answer, "And I just added eight and thirteen". Madison was then asked to estimate the answer to  $\frac{3}{8} + \frac{5}{12}$ , to see if she could use half as a benchmark. Her answer "twelve" was accompanied with the following reasoning: "But eight and twelve are twenty and the three and the five are covering a bit of it and so I took it away." It appeared that adding the numerators and taking away that from the sum of the denominators was her procedure. Applying this whole-number procedure to the previous question, it is clear how she arrived at the answer of two;  $8 + 13$ , which is what she said she did, is 21.  $21 - (7 + 12)$  is 2.

These examples of teachers' knowledge about fractions were sourced from research studies that *interviewed* teachers. They serve to reinforce the evidence that we have presented throughout the synthesis, which clearly illustrates the critical relationship between teacher knowledge enacted in the learning environment and learner outcomes. Specifically, teachers who have strong pedagogical content knowledge and 'connected' knowledge of mathematics, and who also display an awareness of the need to 'connect' with learners' understandings of mathematics, are most likely to occasion quality learning opportunities for diverse learners.

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